PHYSICAL REVIEW E, VOLUME 65, 051701

Flow dissipation effects in a nonlinear nematic fiber

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(Received 2 October 2001; published 15 April 2002)

Dissipative effects due to the presence of hydrodynamic flow in a cylindrical fiber whose cladding is an initially quiescent incompressible nematic liquid crystal are analyzed. An analytic and iterative solution of the nematodynamic equations coupled to the Maxwell's equations describing the propagation of a narrow wave packet of transverse magnetic modes is provided. We derive a generalized nonlinear Schröedinger equation for the amplitude of this propagating wave packet that takes into account the dissipation in the nematic's reorientation and the hydrodynamical effects. For the solitonlike solution of this equation we find that the penetration length and the real part of the nonlinear refraction index increase by a factor of 1.75, with respect to those values obtained in the absence of hydrodynamical flow. The imaginary part remains unaltered.

DOI: 10.1103/PhysRevE.65.051701 PACS number(s): 77.84.Nh, 61.30.Gd, 42.65.−k, 42.81.Dp

The presence of hydrodynamic flow leads to a dynamical anisotropic response of liquid crystals which manifests itself as an effective wave number dependent orientational viscosity [1–3], but its effect on nonlinear optical properties has been less explored. In previous work it has been shown that light induced hydrodynamical motion of a liquid crystal confined in a planar cell, may also produce significant changes in optical properties such as the focal length of the equivalent nonlinear lens and the nonlinear phase change across the cell [4–6].

The lossy and nonlocal effects of the reorientation dynamics of the director when a wave packet of transverse magnetic modes (TM) propagates through a nematic cylindrical fiber in the absence of hydrodynamic flow have been analyzed by using the multiple scales method [7–9]. It has been shown that the dissipation produced by the reorientation, alters the self-focussing, dispersion and diffraction of the wavepacket, leading to a perturbed nonlinear Schrödinger equation (GNLS) for its amplitude [7]. Since it is known that the GNLS equation admits solitonlike solutions [10], the speed, time, and length scales, and penetration length of the optical solitons, as well as the nonlinear index of refraction of the nematic, could be estimated by using experimental values of the relevant parameters [11]. Our purpose in this paper is to describe how the dissipation due to a hydrodynamical flow of the nematic within the fiber affects the penetration length of the wave packet and the nonlinear refraction index of the nematic.

We consider a cylindrical waveguide of length L with an isotropic core of radius a with dielectric constant ϵ_c, and a quiescent, incompressible nematic liquid crystal cladding of radius b, such that $L \gg a,b$. The orientational configuration satisfies the planar axial, boundary strong-anchoring conditions $\hat{n}(r=a,z)=\hat{n}(r=b,z)=\hat{e}_z$, as depicted in Fig. 1. The director is given by $\hat{n}(r,t)=\cos\theta\hat{e}_r+\sin\theta\hat{e}_z$ and the velocity $\vec{v}(\vec{r},t)$ field reads $\vec{v}=v(r,t)\hat{e}_z$, where $\hat{e}_r$ and $\hat{e}_z$ are the unit cylindrical vectors.

In Ref. [12] we derived the coupled dynamics for the TM optical mode, $H_\phi(\vec{r},t)$, and the orientational, $\theta(\vec{r},t)$, field in a cylindrical fiber in a steady state by taking into account explicitly retarded effects, namely

$$\frac{a^2}{c^2}\frac{\partial^2 H_\phi}{\partial t^2} = -\int dt' \frac{\left(\frac{\partial^2 H_\phi}{\partial \zeta^2}+\frac{\partial^2 H_\phi}{\partial x^2}\right)(t-t')}{\epsilon_\perp(\vec{r'},t')} + \frac{\partial^2}{\partial t \partial \zeta}\int dt' \frac{\epsilon_a}{\epsilon_\perp \epsilon_\parallel}(t')\left[-\sin^2\theta\frac{\partial H_\phi}{\partial \zeta} + \sin\theta\cos\theta\frac{1}{x}\frac{\partial}{\partial x}xH_\phi\right](t-t') - \frac{\partial^2}{\partial t \partial x}\int dt' \frac{\epsilon_a}{\epsilon_\perp \epsilon_\parallel}(t')\left[-\sin\theta\cos\theta\frac{\partial H_\phi}{\partial \zeta} + \cos^2\theta\frac{1}{x}\frac{\partial}{\partial x}xH_\phi\right](t-t'), \tag{1}$$

and

$$\frac{dn_i}{dt} = \frac{1}{2}(\partial_k v_i - \partial_i v_k)n_k + \frac{\lambda}{2}(\delta_{il}-n_i n_l)n_k(\partial_k v_l + \partial_l v_k) + \frac{1}{\gamma_1}(\delta_{im}-n_i n_m)\frac{\delta F}{\delta n_m}. \tag{2}$$

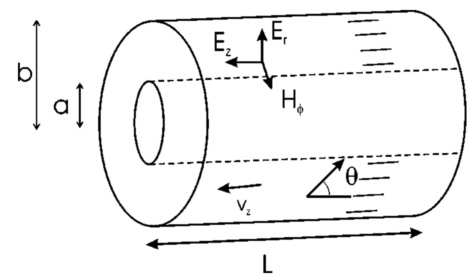

FIG. 1. Schematics of a laser beam propagating through a nematic liquid crystal cylindrical waveguide. The TM modes are shown explicitly.

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Here γ_1 is the reorientational viscosity and δ_{il} is the usual Kronecker delta; $\epsilon_a \equiv \epsilon_\parallel - \epsilon_\perp$ is the dielectric anisotropy where $\epsilon_\parallel$, $\epsilon_\perp$ denote, respectively, the dielectric constants parallel and perpendicular to the long axis of the molecules and $\lambda \equiv -\gamma_2/\gamma_1$ is the ratio of two nematic viscosities. If the reorientation process is isothermal, F in Eq. (2) denotes de Helmholtz free energy that for the present model is [13]

$$F=\pi K\int_V r\,dr\,dz\Bigg[\left(\frac{\sin\theta}{r}-r\sin\theta\frac{\partial\theta}{\partial z}+\cos\theta\frac{\partial\theta}{\partial r}\right)^2 +\left(\cos\theta\frac{\partial\theta}{\partial z}+\sin\theta\frac{\partial\theta}{\partial r}\right)^2+q^2\Bigg\{-\left(\mathcal{E}_r^{a*}\int^t dt'\frac{\partial\mathcal{H}_\phi}{\partial z} -\mathcal{E}_z^{a*}\int^t dt'\frac{1}{r}\frac{\partial(r\mathcal{H}_\phi)}{\partial r}\right)+\mathcal{E}_r^{i*}\int^t dt'\frac{\partial\mathcal{H}_\phi}{\partial z} -\mathcal{E}_z^{i*}\frac{1}{r}\int^t dt'\frac{\partial(r\mathcal{H}_\phi)}{\partial r}\Bigg\}\Bigg]. \tag{3}$$

The dimensionless parameter $q^2 \equiv \epsilon_0 E_0^2 a^2/K$ is equal to the ratio between the electric field energy density and the elastic energy density of the nematic. $K \equiv K_1 = K_2 = K_3$ is the isothermal elastic constant in the equal elastic constants approximation and the asterik (*) indicates complex conjugation. In Eq. (3) $\rightarrow\mathcal{E}^a(\vec{r},t)$ stands for a dimensionless electric field defined by the following nonlocal and retarded relation:

$$\vec{\mathcal{E}}^a(\vec{r},t)=\frac{q}{\epsilon_0}\int dt'\int^t dt''\frac{\epsilon_a}{\epsilon_\perp\epsilon_\parallel}(t''-t')\hat{n}\hat{n}\cdot\vec{\nabla}\times\vec{\mathcal{H}}(\vec{r'},t'), \tag{4}$$

obtained by substitution of the displacement field $\vec{\mathcal{D}}(\vec{r},t)$ in terms of $\vec{\mathcal{H}}(\vec{r},t)$, by using the Ampère-Maxwell law without sources [12]. Here $\zeta \equiv z/a$, $x \equiv r/a$, $H_\phi \equiv \mathcal{H}_\phi/(c\epsilon_0 E_0)$ with $c = 1/\sqrt{\mu_0\epsilon_0}$, where μ_0 and ϵ_0 are the magnetic permeability and dielectric permittivity of free space. The coupling between the director and the TM modes is represented by the term of order q^2 in Eq. (3).

In the present paper we generalize these results by considering the full nematodynamics that must include a coupled set of equations for $\hat{n}$ and $\vec{v}$. Using the formulation in Ref. [14] the equation of motion for a nematic is given by

$$\rho\frac{dv_i}{dt}=-\frac{\partial p}{\partial x_i}-\frac{1}{2}\lambda\frac{\partial}{\partial x_j}(n_ih_j+n_jh_i)-\frac{1}{2}\frac{\partial}{\partial x_j}\left(\Pi_{jl}\frac{\partial}{\partial x_i}n_l +\Pi_{il}\frac{\partial}{\partial x_j}n_l\right)-\frac{1}{2}\frac{\partial^2}{\partial x_j\partial x_l}[(\Pi_{ij}+\Pi_{ji})n_l-\Pi_{il}n_j -\Pi_{jl}n_i]+2\eta_1\frac{\partial}{\partial x_j}v_{ij}+(\eta_3-2\eta_1)\frac{\partial}{\partial x_j}(n_in_lv_{jl} +n_jn_lv_{il})+(\overline{\eta}_2+\eta_1-2\eta_3)\frac{\partial}{\partial x_j}(n_in_jn_ln_mv_{lm}). \tag{5}$$

Here d/dt stands for the material derivative operator and the stress tensor is given by $\Pi_{jl} = \partial F/\partial[(\partial/\partial x_j)n_l] - n_r n_l \partial F/\partial[(\partial/\partial x_j)n_r]$. As usual, the symmetric gradient velocity tensor is $v_{jl} = [(\partial/\partial x_j)v_l + (\partial/\partial x_l)v_j]/2$ and $h_i = (\delta_{im} - n_i n_m)\delta F/\delta n_m$, where $\delta F/\delta n_m$ denotes the variational derivative of F. The kinetic coefficients η_1, $\overline{\eta}_2$, η_3 are the Harvard viscosities and E_0 is the amplitude of the incident field.

We shall now rewrite Eqs. (2), (3), and (5) for the weakly nonlinear TM modes calculated in Ref. [12] for which $q \ll 1$. Since the torques induced by $\mathcal{E}^r$ and $\mathcal{E}^\phi$ are proportional to q and since the magnetic susceptibility is much smaller than the electric one for a thermotropic, it is reasonable to expect that if the initial flow is axial it will remain so as time evolves. Also, the Zocher stresses that could arise from gradients in the magnetic field are negligible in this regime. Furthermore, since the fluid is incompressible and the process is isothermal, the pressure $p = p(\rho,T)$ is constant. Therefore, in the absence of external pressure gradients substitution of the explicit forms for $\hat{n}$ and $\vec{v}$ into Eqs. (2), (5), and (3) leads to

$$\frac{\partial\theta}{\partial t}=\frac{1}{a}\cos^2\theta\frac{dv_z}{dx}-\frac{\sin 2\theta}{4a}\frac{dv_z}{dz}+\frac{1}{\gamma_1}\frac{\delta F}{\delta\theta}, \tag{6}$$

$$\rho\frac{dv_z}{dt}=\frac{1}{a^2}\frac{1}{x}\frac{\partial}{\partial x}x-\frac{\lambda a}{2}\cos 2\theta\frac{\delta F}{\delta\theta}+\frac{K}{2a}\frac{\partial}{\partial x}\cos 2\theta\frac{\partial\theta}{\partial x} +\eta_1\frac{\partial v_z}{\partial x}+(\overline{\eta}_2-3\eta_1)\sin\theta\cos\theta\frac{dv_z}{dx}, \tag{7}$$

with

$$\frac{\delta F}{\delta\theta}=\frac{K}{a^2}\Bigg\{\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial\theta}{\partial x}\right)-\frac{\sin\theta\cos\theta}{x^2} -q^2\Bigg[\frac{\cos 2\theta}{x}\left(\mathcal{E}_z^{a*}\int^t dt'\frac{\partial x\mathcal{H}_\phi}{\partial x}+\mathcal{E}_r^{a*}\int^t dt'\frac{\partial\mathcal{H}_\phi}{\partial\zeta}\right) +\frac{\sin 2\theta}{x^2}\left(-x\mathcal{E}_r^{a*}\int^t dt'\frac{\partial\mathcal{H}_\phi}{\partial\zeta}+\mathcal{E}_z^{a*}\int^t dt'\frac{\partial x\mathcal{H}_\phi}{\partial x}\right)\Bigg]\Bigg\}. \tag{8}$$

The usual procedure to solve Eqs. (6) and (7) is by using the approximations of negligible inertia [1] and minimal coupling [15], where the rapidly varying hydrodynamic velocity is considered to be a slow variable that follows instantaneously the director dynamics. These approximations lead to an amplitude equation for the orientation with an effective viscosity [6]. However, in the present model we have an even faster variable than v_z, namely, the TM mode $H_\phi(\zeta,x,t)$, which couples to both $\theta(\zeta,x,t)$ and $v_z(\zeta,x,t)$, according to Eqs. (1), (6), and (7). Thus, it will be inconsistent to set $dv_z/dt=0$ and instead, since we are considering weakly nonlinear TM modes only, we solve Eqs. (1) and (6) iteratively in powers of q. To this end we assume the following expansions for θ and H_ϕ in powers of q:

$$\theta(\zeta,x,t)=\theta^{(o)}+q^2|A(\Xi,T)U(x,\omega)|^2\theta^{(1)}(\zeta,x,t)+\cdots, \tag{9}$$

$$v_z(\zeta,x,t)=q^2|A(\Xi,T)U(x,\omega)|^2v_z^{(1)}(\zeta,r,t)+\cdots, \tag{10}$$

$$H_\phi(x,\zeta,t)=qU_\phi\left(x,\omega_0+iq\frac{\partial}{\partial T}\right)A(\Xi,T)+q^2U^{(2)}+\text{c.c.}+\cdots. \tag{11}$$

Here $A(\Xi,T)$ is a slowly varying function of the variables $\Xi\equiv q\zeta$ and $T\equiv qt$, which represents the envelope of a narrow wave packet of width $q=(\omega-\omega_0)/\omega_0$ with central frequency ω_0. $\theta^{(n)}$ and $v_z^{(n)}$, with $n=0,1,\ldots$, denote the successive corrections of order n that satisfy the strong-anchoring homeotropic boundary conditions, $\theta(x=1)=\theta(x=b/a)=0$. $U_\phi(x,\omega_0)$ is the linear solution for H_ϕ given by [12,16]

$$U_\phi(x,\omega_0)=J_1^2\left(\sqrt{\epsilon_c\left(\frac{\omega_0 a}{c}\right)^2-\beta^2a^2}\right)\times\sqrt{\frac{\pi}{2\sigma ax}}\exp(-i\beta a\zeta-\sigma ax), \tag{12}$$

with $\sigma=\sqrt{\epsilon_\parallel[\beta^2/\epsilon_\perp-(\omega_0/c)^2]}$ and where $J_1(x)$ is the Bessel function of order 1 and β is the propagation parameter which takes the values given in Table I in Ref. [12]. $U^{(n)}$, $n=2,3,\ldots$ in Eq. (11), are the contributions due to the higher order optical harmonics that might be generated by the nonlinearities of Eqs. (1) and (6). Note that the presence of higher powers of q in Eq. (11), implies that the contribution of the higher order harmonics is smaller than the dominant term that is itself a small amplitude narrow wave packet.

Inserting expression (9) into Eq. (11) and expanding in powers of q it is straightforward to rewrite the latter equation as [12]

$$\hat{L}(\beta,\omega,x)H_\phi+q^2\hat{F}(H_\phi)=0, \tag{13}$$

where the linear and nonlinear operators $\hat{L}$, $\hat{F}$ are defined, respectively, by

$$\hat{L}\equiv\frac{1}{x^2\epsilon_\perp\epsilon_\parallel}\left\{-\epsilon_\perp+x^2\epsilon_\parallel\left[\epsilon_\perp\left(\frac{\omega_0}{c}a\right)^2-(\beta a)^2\right]+x\epsilon_\perp\frac{\partial}{\partial x}+x^2\epsilon_\perp\frac{\partial^2}{\partial x^2}\right\} \tag{14}$$

and

$$\hat{F}\equiv\frac{\epsilon_a|A(\zeta)U(x,\omega)|^2}{x\epsilon_\perp\epsilon_\parallel}i\beta a\left[U\theta^{(1)}(x)+3x\theta^{(1)}(x)\frac{dU}{dx}+Ux\frac{d\theta^{(1)}(x)}{dx}\right]A(\zeta). \tag{15}$$

The governing equation for the first order solution $\theta^{(1)}(\omega,x,\zeta)$ and $v_z^{(1)}(\omega,x,\zeta)$ can be found by inserting Eqs. (9), (10), and (12) into Eq. (6), this leads to

$$-\frac{\gamma_1a^2}{K}\frac{\partial\theta^{(1)}}{\partial t}+\frac{\partial^2\theta^{(1)}}{\theta\zeta^2}+\frac{\partial^2\theta^{(1)}}{\partial x^2}+\frac{1}{x}\frac{\partial\theta^{(1)}}{\theta x}-\frac{\theta^{(1)}}{x^2}+\frac{a\gamma_1}{K}\frac{dv_z^{(1)}}{dx}-\frac{4\epsilon_a\beta a}{\pi^2\epsilon_\perp\epsilon_\parallel}|U(r,t)|^2=0, \tag{16}$$

$$\rho\frac{dv_z^{(1)}}{dt}=\frac{1}{a^2}\frac{1}{x}\frac{\partial}{\partial x}x\left[-\frac{\lambda a}{2}\frac{K}{a^2}\left(\frac{\partial^2\theta^{(1)}}{\theta\zeta^2}+\frac{\partial^2\theta^{(1)}}{\partial x^2}+\frac{1}{x}\frac{\partial\theta^{(1)}}{\theta x}-\frac{\theta^{(1)}}{x^2}-\frac{4\epsilon_a\beta a}{\pi^2\epsilon_\perp\epsilon_\parallel}|U(r,t)|^2\right)+\frac{K}{2a}\frac{\partial^2\theta^{(1)}}{\partial x^2}+\eta_1\frac{dv_z^{(1)}}{dx}\right], \tag{17}$$

where $U(r,t)=\int_0^\infty(\sqrt{\pi}A_2/\sqrt{2x})\exp(-\sigma x)e^{i\omega t}d\omega$. It is convenient to rewrite these equations in the form

$$-\frac{\gamma_1a^2}{K}\frac{\partial e^{i\phi}\theta^{(1)}}{\partial t}+\nabla^2[e^{i\phi}\theta^{(1)}]+\frac{a\gamma_1}{K}e^{i\phi}\frac{dv_z^{(1)}}{dx}=\frac{4\epsilon_a\beta a}{\pi^2\epsilon_\perp\epsilon_\parallel}e^{i\phi}|U(r,t)|^2, \tag{18}$$

$$\rho\frac{de^{i\phi}v_z^{(1)}}{dt}=\frac{1}{a^2}\frac{1}{x}\frac{\partial}{\partial x}x\left[-\frac{\lambda a}{2}\frac{K}{a^2}\left(\nabla^2[e^{i\phi}\theta^{(1)}]-\frac{4\epsilon_a\beta a}{\pi^2\epsilon_\perp\epsilon_\parallel}e^{i\phi}|U(r,t)|^2\right)+\frac{K}{2a}\frac{\partial^2e^{i\phi}\theta^{(1)}}{\partial x^2}+\eta_1\frac{\partial e^{i\phi}v_z^{(1)}}{\partial x}\right]. \tag{19}$$

To solve these equations we define the following Fourier transform:

$$\tilde{\theta}^{(1)}(\vec{\kappa},\omega)=\int d\vec{r}dte^{i\vec{\kappa}\cdot\vec{r}-i\omega t}e^{i\phi}\theta^{(1)}(\vec{r},t), \tag{20}$$

$$\widetilde{v_z}^{(1)}(\vec{\kappa},\omega)=\int d\vec{r}dte^{i\vec{\kappa}\cdot\vec{r}-i\omega t}e^{i\phi}v_z^{(1)}(\vec{r},t), \tag{21}$$

to obtain

$$\left(\rho i\omega+\frac{\eta_1}{a^2}\kappa_r^2\right)\tilde{v}_z^{(1)}(\vec{\kappa},\omega)=\frac{i\kappa_r}{a}\frac{K}{a^2}\left(\frac{1}{2}(\kappa_r^2-\lambda\kappa^2)\kappa^2\tilde{\theta}^{(1)}(\vec{\kappa},\omega)-\frac{4\epsilon_a\beta a}{\pi^2\epsilon_\perp\epsilon_\parallel}S(\vec{\kappa}_1,\omega_1)S^*(\vec{\kappa}_2,\omega_2)\right), \tag{22}$$

and

$$\left(i\frac{\gamma_1 a^2\omega}{K}+\kappa^2\right)\tilde{\theta}^{(1)}(\vec{\kappa},\omega)-i\frac{a\gamma_1}{K}\kappa_r\tilde{v}_z^{(1)}(\vec{\kappa},\omega)=-\frac{4\epsilon_a\beta a}{\pi^2\epsilon_\perp\epsilon_\parallel}S(\vec{\kappa}_1,\omega_1)S^*(\vec{\kappa}_2,\omega_2), \tag{23}$$

where $S(\vec{\kappa},\omega)$ is the Fourier's transform of $e^{i\phi/2}U(r,t)$. Solving Eqs. (22) and (23) for $\tilde{\theta}^{(1)}(\vec{\kappa},\omega)$ we find that

$$\tilde{\theta}^{(1)}(\vec{\kappa},\omega)=-\frac{4\epsilon_a\beta a}{\pi^2\epsilon_\perp\epsilon_\parallel}\int d\omega\, d\vec{\kappa}\frac{S(\vec{\kappa}_1,\omega_1)S^*(\vec{\kappa}_2,\omega_2)}{G(\vec{\kappa},\omega)}\times\delta(\vec{\kappa}-\vec{\kappa}_1-\vec{\kappa}_2)\,\delta(\omega-\omega_1-\omega_2) \tag{24}$$

with

$$G(\vec{\kappa},\omega)^{-1}\equiv\frac{i\omega\rho+[\eta_1+\gamma_1]\frac{\kappa_r^2}{a^2}}{\left(\frac{i\gamma_1 a^2\omega}{K}+\kappa^2\right)\left(i\omega\rho+\frac{\eta_1}{a^2}\kappa_r^2\right)+\frac{\kappa_r^2}{2a^2}\gamma_1(\kappa_r^2-\lambda\kappa^2)}. \tag{25}$$

Note that this expression exhibits explicitly the coupling between the two optical wave vectors $\vec{\kappa}_1$ and $\vec{\kappa}_2$, and the orientational wave vector $\vec{\kappa}$ that for some cases, as the one considered below, can be simplified.

To follow the dynamics of the envelope $A(\Xi,T)$ we substitute Eq. (11) into Eq. (13) and identify the Fourier variables $i\beta a\equiv i\beta_0 a+q\partial/\partial\Xi_1+q^2\partial/\partial\Xi_2$ and $-i\omega\equiv-i\omega_0+q\partial/\partial T$, in consistency with the definition of a narrow wave packet, where $\Xi_n\equiv q^{n-1}\Xi=q^n\zeta$ with $n=1,2,3,\ldots$ are the spatial scales associated with upper harmonics contributions. Expanding the resulting expression up to third order in q, that is,

$$0=\hat{L}\left(i\beta_0 a+q\frac{\partial}{\partial\Xi_1}+q^2\frac{\partial}{\partial\Xi_2}+q^3\frac{\partial}{\partial\Xi_3}-i\omega_0+q\frac{\partial}{\partial T}\right)H_\phi(x,\zeta,t)+q^2\hat{F}(H_\phi(x,\zeta,t)), \tag{26}$$

and grouping together terms of the same order in q, we get equations for $A(\Xi,T)$ for each of the spatial scales Ξ,Ξ_1,Ξ_2. If the equations for the lower scales Ξ and $\Xi_1$are inserted into the equation for Ξ_2 we arrive at the NLS equation [12],

$$q^3{:}\,2\frac{\partial A}{\partial\Xi_2}+ia\frac{d^2\beta}{d\omega^2}\frac{\partial^2 A}{\partial T^2}+i\overline{n_2}A|A|^2=0, \tag{27}$$

where the dimensionless refraction index $\overline{n_2}\equiv Kn_2/\epsilon_0 a^2$ is given by

$$\overline{n_2}=\frac{\epsilon_a}{\epsilon_\parallel}\left[\left\langle|U_\phi(x,\omega_0)|^2U_\phi(x,\omega_0)\left(\frac{\theta^{(1)}(x)}{x}+\frac{d\theta^{(1)}(x)}{dx}\right),\ U_\phi(x,\omega_0)\right\rangle+3\left\langle\theta^{(1)}(x)|U_\phi(x,\omega_0)|^2\frac{dU_\phi(x,\omega_0)}{dx},\ U_\phi(x,\omega_0)\right\rangle\right]\Big/\langle U_\phi(x,\omega_0),U_\phi(x,\omega_0)\rangle. \tag{28}$$

Here the angular brackets denote integration over x from $x=1$ to $x=b/a$. To calculate this integral we must first obtain $\theta^{(1)}(x,\omega)$, which stems from the inverse Fourier transform of Eq. (24). For the case of the narrow optical wave packet considered here we have to take into account the two possible couplings, namely, $\vec{\kappa}_1+\vec{\kappa}_2=2\vec{\kappa}_i$ with $\omega_1-\omega_2=0$ or $\omega_1+\omega_2=0$. Thus, Eq. (24) may be approximated as

$$\tilde{\theta}^{(1)}(\vec{\kappa}_i,\omega)=-\frac{4\epsilon_a\beta a}{\pi^2\epsilon_\perp\epsilon_\parallel}\left[\frac{\left(i\alpha_2^{-1}+\left[1+\frac{\gamma_1}{\eta_1}\right]\kappa_r^2\right)|S(\vec{\kappa}_i,\omega_0)|^2}{4\left[(i\alpha_1^{-1}+\kappa_i^2)(i\alpha_2^{-1}+\kappa_{ir}^2)+\frac{\kappa_r^2}{2}\frac{\gamma_1}{\eta_1}(\kappa_{ir}^2-\lambda\kappa_i^2)\right]}+\frac{[\eta_1+\gamma_1]|S(\vec{\kappa}_i)|^2}{4\left[\left(\eta_1+\frac{\gamma_1}{2}-\frac{\gamma_1}{2}\lambda\right)\kappa_r^2+\left(\eta_1+\frac{\gamma_1\lambda}{2}\right)\kappa_z^2\right]}\right], \tag{29}$$

where we have taken $\epsilon(k_0,-\omega_0)=\epsilon(k_0,\omega_0)$ by assuming absorption to be negligible and where $\alpha_1 \equiv K/\gamma_1 a^2\omega_0$ and $\alpha_2 \equiv \eta_1/\omega_0\rho a^2$. It is straightforward to show that α_1 and α_2 are small by substituting numerical values for a typical nematic, namely, $K=10^{-11}$N, $\gamma_1=95\times10^{-3}$kg s^{-1} m^{-1}, $\eta_1=121\times10^{-3}$ kg s^{-1} m^{-1}, $\rho=10^3$ kg/m^3 and an optical frequency, $\omega_0=3.8\times10^{15}$ rad/s. This yields the values $\alpha_1=1.3\times10^{-9}$ and $\alpha_2=9.2\times10^{-8}$, hence the first term of Eq. (29) has a negligible real part and is much smaller than the second term. Taking the inverse spatial Fourier transform of the latter equation we obtain

$$\theta^{(1)}(x,\omega)=i\alpha_1\frac{2\epsilon_a\beta a}{\pi\epsilon_\perp\epsilon_\parallel}\frac{J_1^2\left(\sqrt{\epsilon_c\left(\frac{\omega_0 a}{c}\right)^2-\beta^2a^2}\right)}{x}\times\exp(-2\sigma a x)+\frac{[\eta_1+\gamma_1]}{\left(\eta_1+\frac{\gamma_1}{2}-\frac{\gamma_1}{2}\lambda\right)}\times\frac{\beta a\epsilon_a J_1^2\left(\sqrt{\epsilon_c\left(\frac{\omega_0 a}{c}\right)^2-\beta^2a^2}\right)}{\pi\epsilon_\perp\epsilon_\parallel x(a^2-b^2)}\times\{(a^2-b^2)e^{\sigma a(1-x)}+(b^2-x^2a^2)+e^{\sigma(a-b)}a^2(1-x^2)\},\qquad(30)$$

where the real part of $\theta^{(1)}(x,\omega)$ is proportional to the one obtained in the absence of dissipation [12] that satisfies the hard anchoring homeotropic boundary conditions given above. Inserting $\theta^{(1)}(x,\omega)$ into Eq. (28) yields

$$\overline{n}_2=\frac{1}{4}\epsilon_a^2\beta a^3\frac{[\eta_1+\gamma_1]}{\left(\eta_1+\frac{\gamma_1}{2}-\frac{\gamma_1}{2}\lambda\right)}\times J_1^4\left(\sqrt{\epsilon_c\left(\frac{\omega_0 a}{c}\right)^2-\beta^2a^2}\right)e^{-\sigma b+2\sigma a}\times\frac{-ae^{-3\sigma b}+ae^{\sigma(a-4b)}+be^{-\sigma(4a-b)}-be^{-3\sigma a}}{\pi\epsilon_\parallel^2 b(a^2-b^2)\epsilon_\perp(-e^{-2\sigma b}+e^{-2\sigma a})}-\frac{5i\epsilon_a\alpha\beta a^2}{\epsilon_\perp\epsilon_\parallel}J_1^4\left(\sqrt{\epsilon_c\left(\frac{\omega_0 a}{c}\right)^2-\beta^2a^2}\right)\times\frac{e^{-6b\sigma}-\left(\frac{a}{b}\right)^2e^{-6\sigma a}}{e^{-2b\sigma}-\frac{a}{b}e^{-2\sigma a}},\qquad(31)$$

which is a complex nonlinear refraction index, $\overline{n_2}=\overline{n_2^r}+i\overline{n_2^i}$. It should be noticed, on the one hand, that the direct comparison of the real part $\overline{n_2^r}$ and the ones obtained for $\overline{n_2}$ in the absence of dissipation and hydrodynamical effects, given by Eqs. (40) and (21) of Refs. [7,12], respectively, show that the only difference between them is the factor $(\eta_1+\gamma_1)/[\eta_1+\gamma_1/2-(\gamma_1/2\lambda)]$. For the material parameter values given above this factor has the value 1.75. On the other hand, the imaginary part $\overline{n_2^i}$ of $\overline{n_2}$ remains the same as that found in Ref. [12]. Thus, the values of $\overline{n_2^i}$ as given in Table I of Ref. [12] do not change with the hydrodynamical effects, while the values of $\overline{n_2^r}$ from the same table have to be increased by 1.75.

Similarly, as in absence of hydrodynamical effects [7], we can take into account this lossy contribution as a perturbative term in the NLS equation (27). Indeed, Eq. (27) can be rewritten as the GNLS equation

$$\frac{\partial A}{\partial\overline{\Xi}^2}+i\frac{\partial^2A}{\partial\overline{T}^2}-i|A|^2A-\overline{\alpha}|A|^2A=0,\qquad(32)$$

with $\overline{\alpha}=\overline{n_2^i}/\overline{n_2^r}$. By considering the last term as a perturbation, it was found the soliton-type solution [10,12]

$$A=2\eta\,\mathrm{sech}[\overline{T}-\overline{Z}(dk/d\omega)Z_0/T_0]\times\exp[ik(\omega_0)Z_0\overline{Z}-i\omega_0T_0\overline{T}],\qquad(33)$$

where η is given by $\eta(\overline{Z})=1/\sqrt{1+16\overline{\alpha}\overline{Z}/3}$, and where the initial condition $\eta(\overline{Z}=0)=1$ has been imposed. This expression shows that dissipation due to flow reorientation makes the soliton amplitude η decrease with the distance; η falls half its initial amplitude A_0 when the soliton has traveled the distance $\overline{Z}_a=9/(16\overline{\alpha})$.

Let us now estimate the changes in the length and time scales, $Z_0=2/\overline{n_2^r}A_0^2$, $T_0^2=(a/\overline{n_2^r}A_0^2)d^2\beta/d\omega_0^2$, of this pulse owing to the presence of hydrodynamic flow. For a 500 mW laser at $\lambda=0.5$ μm with a beam waist of 10 μm, the field amplitude is $A_0^2=1.9\times10^6$ V/m. Then, by using the material constants given above and for the mode with $\beta a=229.59$, this leads to the following spatial and temporal scales for the pulse, $Z_0=2.4\times10^{-5}$ m and $T_0=0.12\times10^{-12}$ s.

Finally, because $\overline{n_2^r}$ in $\overline{\alpha}$ increases by a factor of 1.75, the characteristic distance over which the soliton loses half its initial amplitude A_0, given by $z_d=Z_0Z_a$, leads to a larger value given by $z_d^{nem}=1.92K$m due to hydrodynamical effects [5].

Summarizing, we have derived a generalized nonlinear Schröedinger equation for the amplitude of a wave packet of TM modes propagating through a cylindrical nematic waveguide by taking into account dissipation in the nematic's reorientation and hydrodynamical effects. For the solitonlike solution of this equation we found an increase of a factor 1.75 in both, the penetration length and the nonlinear refraction index.

We acknowledge partial financial support from Grant No. DGAPA UNAM IN105797, México.

[1] E. Guyon, R. Meyer, and J. Salán, Mol. Cryst. Liq. Cryst. **54**, 261 (1979)
[2] Y.W. Hui, M.R. Kuzma, M. San Miguel, and M.M. Labes, J. Chem. Phys. **83**, 288 (1985).
[3] R.F. Rodríguez, M. San Miguel, and F. Sagués, Mol. Cryst. Liq. Cryst. **199**, 393 (1991).
[4] E.F. Carr, Mol. Cryst. Liq. Cryst. **34**, L159 (1977).
[5] R.F. Rodríguez, P. Ortega, and R. Díaz-Uribe, Physica A **230**, 118 (1996).
[6] R.F. Rodríguez and P. Ortega, Mol. Cryst. Liq. Cryst. Sci. Technol., Sect. A **222**, 45 (1992).
[7] R.F. Rodríguez and J.A. Reyes, Opt. Commun. **197**, 103 (2001).
[8] A. C. Moloney and J. V. Newell, *Nonlinear Optics* (Addison Wesley, New York, 1992).
[9] R.F. Rodríguez and J.A. Reyes, J. Mol. Liq. **71**, 115 (1997).
[10] D.J. Kaup and A.C. Newell, Proc. R. Soc. London, Ser. A **361**, 413 (1978).
[11] Shu-Hsia Chen and Tien-Jung Chen, Appl. Phys. Lett. **64**, 1893 (1994).
[12] J.A. Reyes and R.F. Rodríguez, Physica D **101**, 333 (2000).
[13] F.C. Frank, Discuss. Faraday Soc. **25**, 19 (1958).
[14] L.D. Landau, E.M. Lifshitz, A.M. Kosevich, and L.P. Pitaevskii, *Theory of Elasticity* (Pergamon Press, New York, 1981).
[15] A.J. Hurd, S. Fraden, F. Lonberg, and R.B. Meyer, J. Phys. (France) **46**, 905 (1985).
[16] D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1984).